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Three-dimensional black holes, gravitational solitons, kinks and wormholes for BHT massive gravity

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Abstract: The theory of massive gravity in three dimensions recently proposed by Bergshoeff, Hohm and Townsend (BHT) is considered. At the special case when the theory admits a unique maximally symmetric solution, a conformally flat solution that contains black holes and gravitational solitons for any value of the cosmological constant is found. For negative cosmological constant, the black hole is characterized in terms of the mass and the "gravitational hair" parameter, providing a lower bound for the mass. For negative mass parameter, the black hole acquires an inner horizon, and the entropy vanishes at the extremal case. Gravitational solitons and kinks, being regular everywhere, can be obtained from a double Wick rotation of the black hole. A wormhole solution in vacuum that interpolates between two static universes of negative spatial curvature is obtained as a limiting case of the gravitational soliton with a suitable identification. The black hole and the gravitational soliton fit within a set of relaxed asymptotically AdS conditions as compared with the one of Brown and Henneaux. In the case of positive cosmological constant the black hole possesses an event and a cosmological horizon, whose mass is bounded from above. Remarkably, the temperatures of the event and the cosmological horizons coincide, and at the extremal case one obtains the analogue of the Nariai solution, $dS_2 \times S^1$. A gravitational soliton is also obtained through a double Wick rotation of the black hole. The Euclidean continuation of these solutions describes instantons with vanishing Euclidean action. For vanishing cosmological constant the black hole and the gravitational soliton are asymptotically locally flat spacetimes. The rotating solutions can be obtained by boosting the previous ones in the $t-\phi$ plane.

Keywords: Field Theories in Lower Dimensions, Black Holes, Classical Theories of Gravity, Space-Time Symmetries

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Contents

1 Introduction

As shown by Brown and Henneaux, General Relativity with negative cosmological constant in three dimensions appeared as the first example of a field theory admitting a classical central charge given by [1]

$$c = \frac{3l}{2G}\,, \tag{1.1}$$

where l is the AdS radius, and G is the Newton constant. This is possible due to the enhancement of the asymptotic symmetries from $\mathrm{SO}(2,2)$ to the infinite dimensional conformal group in two dimensions. Remarkably, the AdS/CFT correspondence [2] was foreseen during the 80's within this context.

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The asymptotic behavior of the metric is given by [1]

$$\begin{aligned}\Delta g_{rr} &= f_{rr} r^{-4} + O(r^{-5}) \ , \\ \Delta g_{rm} &= f_{rm} r^{-3} + O(r^{-4}) \ , \\ \Delta g_{mn} &= f_{mn} + O(r^{-1}) \ .\end{aligned} \tag{1.2}$$

Here $f_{\mu\nu} = f_{\mu\nu}(t, \phi)$, and the indices have been split as $\mu = (r, m)$, where m includes the time and the angle. The asymptotic metric is written as $g_{\mu\nu} = \bar{g}_{\mu\nu} + \Delta g_{\mu\nu}$, where $\Delta g_{\mu\nu}$ corresponds to the deviation from the AdS metric,

$$d\bar{s}^2 = -(1 + r^2/l^2)dt^2 + (1 + r^2/l^2)^{-1}dr^2 + r^2 d\phi^2 \ . \tag{1.3}$$

The asymptotic conditions (1.2) map into themselves under diffeomorphisms of the form

$$\begin{aligned}\eta^+ &= T^+ + \frac{l^2}{2r^2}\partial_-^2 T^- + \cdots \\ \eta^- &= T^- + \frac{l^2}{2r^2}\partial_+^2 T^+ + \cdots \\ \eta^r &= -\frac{r}{2}\left(\partial_+ T^+ + \partial_- T^-\right) + \cdots ,\end{aligned} \tag{1.4}$$

where $T^\pm = T^\pm(x^\pm)$, with $x^\pm = \frac{t}{l} \pm \phi$, and the dots stand for lower order terms that do not contribute to the surface integrals. Thus, the boundary conditions (1.2) are invariant under two copies of the Virasoro group, generated by $T^+(x^+)$ and $T^-(x^-)$. The Poisson brackets of the canonical generators, defined by surface integrals at infinity that depend on the metric and its derivatives, reproduces then two copies of the Virasoro algebra with central charge given by (1.1).

It is known that there are instances where the asymptotic behavior (1.2) for pure gravity with localized matter fields can be relaxed in order to accommodate solutions of physical interest, without spoiling the asymptotic symmetries (1.4).

This occurs for General Relativity with negative cosmological constant coupled to scalar fields of mass within the range

$$m_*^2 \leq m^2 < m_*^2 + \frac{1}{l^2} \ , \tag{1.5}$$

where

$$m_*^2 = -\frac{(d-1)^2}{4l^2} \ , \tag{1.6}$$

defines the Breitenlohner-Freedman bound [3]. In this case, the scalar field possesses a very slow fall-off at infinity that generates a strong back reaction in the metric, so that the standard AdS asymptotic conditions of [1, 4, 5] have to be relaxed. As a consequence, the charges do not only depend on the metric and its derivatives, but acquire an explicit contribution from the matter field. The role of this additional contribution is to cancel the divergences coming from the purely gravitational contribution in order to render the surface integrals defining the charges to be finite. This was investigated at length in [6–8] for any dimension (see also [9, 10]).

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As a consequence of the softening of the boundary conditions, the space of admissible solutions is enlarged so as to include hairy black holes [6, 9, 11][1], solitons and instantons [14].

One could envisage that a generic effect of relaxing the asymptotic conditions in gravitation is the allowance of hairy solutions, since it is known that this effect extends beyond General Relativity with scalar fields. Indeed, it has been recently shown in [21] that a similar phenomenon occurs for topologically massive gravity [15], where the action of Einstein gravity with negative cosmological constant in three dimensions is supplemented by the Lorentz-Chern-Simons term. For the range $0 < |\mu l| \leq 1$, where μ is the topological mass parameter, it was found that topologically massive gravity admits a set of relaxed asymptotically AdS boundary conditions allowing the inclusion of the AdS waves solutions discussed in refs. [16–18]. In the case of $0 < |\mu l| < 1$, one can see that even though the asymptotic conditions are relaxed with respect to the standard ones (1.2) the charges acquire the same form as if one had considered the Brown-Henneaux boundary conditions. This is because the diverging pieces associated with the slower fall-off cancel out, so that the charges acquire no correction involving the terms associated with the relaxed behavior. As a consequence, the terms with slower fall-off, which cannot be gauged away, can be seen as defining a kind of "hair"[2].

It is natural then wondering whether these effects could also appear for the theory of massive gravity that has been recently proposed by Bergshoeff, Hohm and Townsend (BHT) [24]. The action for the BHT massive gravity theory is given by

$$I_{\mathrm{BHT}} = \frac{1}{16\pi G}\int d^3x\sqrt{-g}\left[R - 2\lambda - \frac{1}{m^2}K\right] , \tag{1.7}$$

where K stands for a precise combination of parity-invariant quadratic terms in the curvature:

$$K := R_{\mu\nu}R^{\mu\nu} - \frac{3}{8}R^2 \ . \tag{1.8}$$

The field equations are then of fourth order and read

$$G_{\mu\nu} + \lambda g_{\mu\nu} - \frac{1}{2m^2}K_{\mu\nu} = 0\,, \tag{1.9}$$

where

$$K_{\mu\nu} := 2\nabla^2 R_{\mu\nu} - \frac{1}{2}\left(\nabla_\mu\nabla_\nu R + g_{\mu\nu}\nabla^2 R\right) - 8R_{\mu\rho}R^{\rho}{}_{\nu} + \frac{9}{2}RR_{\mu\nu} + g_{\mu\nu}\left[3R^{\alpha\beta}R_{\alpha\beta} - \frac{13}{8}R^2\right] , \tag{1.10}$$

[1]Remarkably, the phase transition found in [11] allowing the decay of the black hole in vacuum into the hairy black hole, according to the proposal of [12], has been shown to be the gravity dual of the transition to superconductivity of a gapless superconductor [13].

[2]As pointed out in [19], the case $|\mu l| = 1$, known as the chiral point, enjoys remarkable properties. In particular, in order to accommodate the new solutions, one must allow for logarithmic terms in the asymptotic behaviour of the metric [20, 21], and these logarithmic terms make both sets of Virasoro generators non-zero, even though one of the central charges vanishes [21]. This has also been verified in [22, 23]. Therefore, exceptionally in this case, since the terms with slower fall-off do contribute to the charges, they are not suitably regarded as "hair".

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fulfills[3] $K = g^{\mu\nu}K_{\mu\nu}$. Remarkably, the BHT massive gravity theory was shown to be equivalent at the linearized level to the (unitary) Fierz-Pauli action for a massive spin-2 field [24]. The unitarity of the BHT theory has been revisited in [25]. Exact solutions have also been found, including warped AdS black holes [26] and AdS waves [27, 28]. Further aspects of the BHT theory have been explored in [29, 30].

As pointed out in [24], generically the theory admits solutions of constant curvature ($R^{\mu\nu}_{\alpha\beta} = \Lambda\delta^{\mu\nu}_{\alpha\beta}$) with two different radii, determined by

$$\Lambda_\pm = 2m\left(m \pm \sqrt{m^2-\lambda}\right) . \tag{1.11}$$

This means that at the special case defined by

$$m^2 = \lambda\,, \tag{1.12}$$

for which $\Lambda_+ = \Lambda_-$, the theory possesses a unique maximally symmetric solution of fixed curvature given by

$$\Lambda = 2\lambda = 2m^2 . \tag{1.13}$$

In this sense, the behavior of the BHT theory is reminiscent to the one of the Einstein-Gauss-Bonnet (EGB) theory, which could be regarded as a higher-dimensional cousin of the same degree (but of lower order). This can be seen as follows:

In $d > 4$ dimensions, the action for the EGB theory reads

$$I_{\rm EGB} = \kappa\int d^dx\sqrt{-g}\left(R - 2\lambda + \alpha\left(R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} - 4R_{\mu\nu}R^{\mu\nu} + R^2\right)\right) , \tag{1.14}$$

where the quadratic terms appear in a precise combination so that the field equations are of second order [31]. Generically, the EGB theory admits two solutions of constant curvature $R^{\mu\nu}_{\alpha\beta} = \Lambda\delta^{\mu\nu}_{\alpha\beta}$, whose radii are fixed according to

$$\Lambda_\pm = \frac{1}{2\tilde{\alpha}}\left[1 \pm \sqrt{1+4\tilde{\alpha}\tilde{\lambda}}\right] , \tag{1.15}$$

with

$$\tilde{\alpha} := (d-3)(d-4)\alpha\ ;\ \tilde{\lambda} := \frac{2\lambda}{(d-1)(d-2)} . \tag{1.16}$$

Hence, at the special case for which $\Lambda_+ = \Lambda_-$, given by

$$1 + 4\tilde{\alpha}\tilde{\lambda} = 0\,, \tag{1.17}$$

the theory possesses a unique maximally symmetric solution [33].

The static and spherically symmetric solution was found by Boulware and Deser [34], and it is given by

$$ds^2 = -f^2_\pm(r)dt^2 + \frac{dr^2}{f^2_\pm(r)} + r^2d\phi^2, \tag{1.18}$$

[3] Here $\nabla^2 := \nabla^\mu\nabla_\mu$, and for the spacetime signature we follow the "mostly plus" convention.

with

$$f_{\pm}^{2}(r)=1+\frac{r^{2}}{2\tilde{\alpha}}\left[1\pm\sqrt{1+4\tilde{\alpha}\,\tilde{\Lambda}+\frac{\mu}{r^{d-1}}}\right]\ . \tag{1.19}$$

Thus, for a generic choice of the Gauss-Bonnet coupling α, the solution possesses two branches, each of them approaching to the maximally symmetric solution at infinity according to

$$f_{\pm}^{2}(r)=\Lambda_{\pm}r^{2}-\frac{\mu_{\pm}}{r^{d-3}}+\cdots, \tag{1.20}$$

Note that for the generic case, the asymptotic behavior has the same fall-off as the one for the Schwarzschild-(A)dS solution of General Relativity (GR) in d dimensions. Nevertheless, for the special case (1.17), the solution has the following fall-off

$$f^{2}(r)=\Lambda r^{2}-\frac{\mu}{r^{\frac{d-5}{2}}}+\cdots \tag{1.21}$$

which is slower than the one for the Schwarzschild-(A)dS solution ($O(r^{3-d})$). One may then fear that the surface integrals at infinity defining the conserved charges blow up. However, as shown in [33], the conserved charges have to be computed from scratch and they turn out to be finite. It was also shown that the black hole fits within a relaxed set of asymptotic conditions possessing the same asymptotic symmetries as for GR (see also [35]).

As it occurs for their three-dimensional counterparts, the consequence of relaxing the asymptotic conditions for the EGB theory is to enlarge the space of admissible solutions so as to include wormholes in vacuum [36–39], gravitational solitons [40] and rotating spacetimes [41].

Thus, one may naturally expect that a similar behavior occurs for the BHT massive gravity theory. The purpose of this paper is to show that this is indeed the case.

2 BHT massive gravity at the special case $m^2 = \lambda$

The field equations of the BHT massive gravity theory (1.9), at the special case $m^2=\lambda$, admit the following exact Euclidean solution:

$$ds^{2}=\left(-\Lambda r^{2}+br-\mu\right)d\psi^{2}+\frac{dr^{2}}{-\Lambda r^{2}+br-\mu}+r^{2}d\varphi^{2}\,, \tag{2.1}$$

where b and μ are integration constants, and $\Lambda:=2\lambda$. When the constant b is switched on, as it is apparent from (2.1), as $r\to\infty$, the Riemann curvature approaches to a constant ($R_{\alpha\beta}^{\mu\nu}\to\Lambda\delta_{\alpha\beta}^{\mu\nu}$). The Ricci scalar of this metric is given by

$$R=6\Lambda-\frac{2b}{r}\ . \tag{2.2}$$

As it was shown in [42], the metric (2.1) is conformally flat, and hence also corresponds to a solution of conformal gravity in three dimensions, as well as for the BHT theory supplemented by the gravitational Lorentz-Chern-Simons form.

As it is shown below, this metric describes instantons for a suitable range of the coordinates and of the parameters b and μ. Furthermore, it is possible to perform different

Wick rotations so that the corresponding metric of Lorentzian signature describes either asymptotically (A)dS or asymptotically locally flat black holes, as well as gravitational solitons and kinks. Further interesting solutions, including wormholes in vacuum can also be obtained as limiting cases of the black holes and the gravitational solitons. The corresponding rotating solutions can then be obtained by boosting the previous ones in the "$t-\phi$" plane.

The different cases are examined according to the sign of the cosmological constant.

3 Negative cosmological constant

3.1 Black hole

In the case of negative cosmological constant, $\Lambda := -\frac{1}{l^2}$, a Lorenzian solution of the field equations (1.9) is obtained from (2.1), making $\psi \to it$ and $\varphi = \phi$. The metric is given by

$$ds^2 = -\left(\frac{r^2}{l^2} + br - \mu\right) dt^2 + \frac{dr^2}{\frac{r^2}{l^2} + br - \mu} + r^2 d\phi^2\,, \tag{3.1}$$

and for the range of coordinates $-\infty < t < +\infty$, $0 \le \phi < 2\pi$, it describes asymptotically AdS black holes provided the lapse function g_{tt} admits a positive real root. In terms of the corresponding roots, $r_+ > r_-$, the metric reads

$$ds^2 = -\frac{1}{l^2}(r-r_+)(r-r_-)dt^2 + \frac{l^2 dr^2}{(r-r_+)(r-r_-)} + r^2 d\phi^2\,, \tag{3.2}$$

where

$$b = -\frac{1}{l^2}(r_+ + r_-)\,, \tag{3.3}$$

$$\mu = -\frac{r_+ r_-}{l^2}\,. \tag{3.4}$$

The mass of this solution, measured with respect to an AdS background ($b = 0$, $\mu = -1$) is given by

$$M = \frac{1+\mu}{4G}\,. \tag{3.5}$$

Remarkably, as the mass is exclusively parametrized in terms of the integration constant μ, the constant b can be regarded as a sort of "gravitational hair". Indeed, one can prove that the black hole has no additional global charges generated by the asymptotic symmetries. This is shown in section 3.3.

In the case of $b = 0$, the solution reduces to the static BTZ black hole [43, 44], while when the gravitational hair parameter is switched on ($b \neq 0$), as it can be seen from (2.2), the geometry develops a curvature singularity at the origin ($r = 0$). According to the sign of b, the singularity can be surrounded by one or two horizons for a suitable range of the mass:

- $b > 0$: When the parameter b is positive, as the root r_- becomes negative, there is a single even horizon located at $r = r_+$, provided the mass parameter fulfills $\mu \ge 0$.

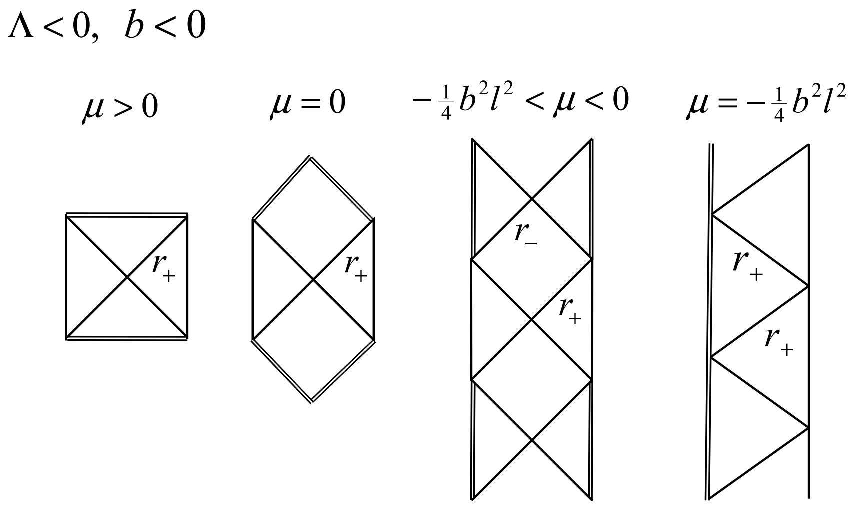

Figure 1. Causal structure of the asymptotically AdS black holes with negative b.

· For $\mu > 0$ the event horizon surrounds singularity at the origin which is spacelike.

· The bound is saturated for $\mu = 0$, and the horizon coincides with the singularity at the origin which becomes null.

- $b < 0$: For negative b, the singularity is surrounded by an event horizon provided the mass parameter is bounded from below by a negative quantity, proportional to the square of the product of the parameter b and the AdS radius l, i.e.,

$$\mu \geq -\frac{1}{4}b^2l^2 \ . \tag{3.6}$$

  · For $\mu > 0$ there is a single event horizon at $r = r_+$, enclosing the spacelike singularity at the origin.

  · In the case of $\mu = 0$ an inner horizon appears at the origin, on top of the singularity which is null.

  · For the range $-\frac{1}{4}b^2l^2 < \mu < 0$, the singularity at the origin becomes timelike and it is enclosed by an inner Cauchy horizon at $r = r_-$, which is surrounded by an event horizon located at $r = r_+$.

  · The bound (3.6) is saturated in the extremal case $\mu = -\frac{1}{4}b^2l^2$, for which both horizons coincide ($r_+ = r_- = -bl^2/2$) enclosing the timelike singularity at the origin.

Thus, switching on the gravitational hair parameter b manifests directly on the causal structure of the black hole, as it is depicted in figures 1 and 2.

The effect of the gravitational hair can also seen as follows. Let us consider a static BTZ black hole of fixed mass:

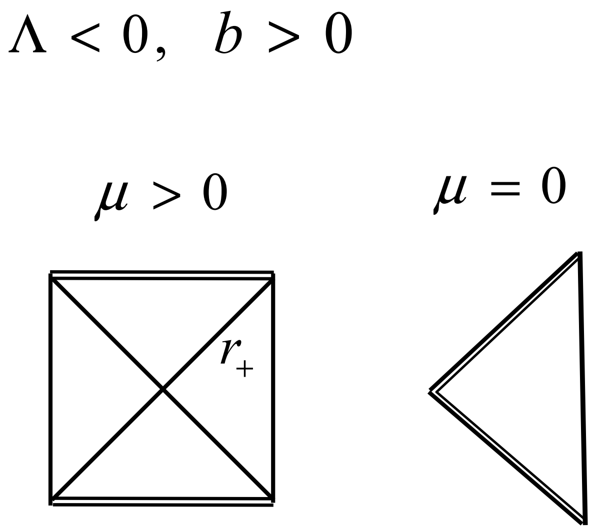

Figure 2. Penrose diagrams of the black holes with negative cosmological constant and positive b.

· Adding positive gravitational hair ($b > 0$) amounts to shrink the black hole horizon; while keeping $b > 0$ fixed, the black hole mass is still bounded as $\mu \geq 0$.

· In the case of adding negative hair ($b < 0$) the black hole horizon increases, and the ground state of the solution changes. This means that while keeping $b < 0$ fixed, the black hole mass is now allowed to be negative up to certain extent, since it is bounded in terms of the gravitational hair parameter according to (3.6), which determines the size of the extremal black hole.

It is worth pointing out that the space of allowed solutions enlarges once the gravitational hair is switched on, since the asymptotic behavior of the metric is relaxed as compared with the one of General Relativity.

3.2 Asymptotically AdS boundary conditions with relaxed behavior

The suitable set of asymptotically AdS conditions that contains the black hole solution (3.1) possesses a relaxed behavior as compared with the one of Brown and Henneaux, given by (1.2). The deviation with respect to the AdS metric (1.3) in our case is given by

$$\begin{aligned}\Delta g_{rr} &= h_{rr}\, r^{-3} + f_{rr}\, r^{-4} + \dots\,, \\ \Delta g_{rm} &= h_{rm}\, r^{-2} + f_{rm}\, r^{-3} + \dots\,, \\ \Delta g_{mn} &= h_{mn}\, r + f_{mn} + \dots\,,\end{aligned} \tag{3.7}$$

where $f_{\mu\nu}$ and $h_{\mu\nu}$ depend only on the time and the angle, but not on r. Here the f-terms correspond to the deviation from the AdS metric proposed by Brown and Henneaux (for General Relativity), while the h-terms take into account the relaxation of the standard boundary conditions that is required in order to include the black hole solution (3.1) with slower fall-off.

The asymptotic symmetry group associated to (3.7) contains the standard two copies of the Virasoro group, generated by the diffeomorphisms η in eq. (1.4), and it is augmented to a semi-direct product by the additional asymptotic symmetries generated by

$$\zeta = Y(x^+, x^-)\partial_r \ . \tag{3.8}$$

As it is shown in the next subsection, conserved charges as surface integrals at infinity exist and they turn out to be finite for the relaxed asymptotic conditions proposed in eq. (3.7). The corresponding central charge is then found to be twice the value found by Brown and Henneaux for GR, i.e.,

$$c = \frac{3l}{G} \ . \tag{3.9}$$

3.3 Conserved charges as surface integrals at infinity

Despite the fact that the asymptotic conditions (3.7) are relaxed by additional terms that grow instead of decaying as one approaches to infinity, one can see that they are mild enough in the sense that finite charges as surface integrals can be consistently constructed through standard perturbative methods. Here we follow the approach of Abbot and Deser [45] to construct conserved charges for asymptotically AdS spacetimes, which has been extended to the case of gravity theories with quadratic terms in the curvature by Deser and Tekin [46]. The conserved charges for the BHT theory can be written as

$$Q_{\mathrm{DT}}(\xi) = \left(1 + \frac{\Lambda}{2m^2}\right) Q_{\mathrm{AD}}(\xi) - \frac{\Lambda}{m^2} Q_K(\xi) \,, \tag{3.10}$$

So that in the limit $m^2 \to \infty$ one recovers the standard expression for GR. Note that the quadratic terms in the action (1.7) change the factor in front of the Abbott-Deser charges, given by $Q_{\mathrm{AD}}(\xi)$, and contribute with an additional piece given by $Q_K(\xi)$. The precise definition of the surface integrals, $Q_{\mathrm{AD}}(\xi)$ and $Q_K(\xi)$, can be extracted from refs. [46, 47].

Evaluating the surface integrals appearing in the Deser-Tekin charges (3.10) on the asymptotic conditions (3.7), for the asymptotic symmetries generated by $\xi = \eta + \zeta$, where η and ζ are given by eqs. (1.4) and (3.8), respectively, one obtains

$$\begin{aligned} Q_{\mathrm{AD}}(\xi) = -\frac{1}{32\pi G l^3} \int d\phi \, \{ & T^+(4l^2 \left(f_{+-} - f_{++}\right) - f_{rr}) + T^-(4l^2 \left(f_{+-} - f_{--}\right) - f_{rr}) \\ & - r \left[\left(T^+ + T^-\right)\left(h_{rr} - 2l^2 h_{+-}\right) + 2l^2 \left(T^+ h_{++} + T^- h_{--}\right) \right] \} \end{aligned}$$
$$, \tag{3.11}$$

$$\begin{aligned} Q_K(\xi) = -\frac{1}{32\pi G l^3} \int d\phi \, \{ & T^+(4l^2 f_{+-} - f_{rr}) + T^-(4l^2 f_{+-} - f_{rr}) \\ & - r \left[\left(T^+ + T^-\right)\left(h_{rr} - 2l^2 h_{+-}\right) + 2l^2 \left(T^+ h_{++} + T^- h_{--}\right) \right] \} \ , \end{aligned} \tag{3.12}$$

so that the full charge reads

$$
\begin{aligned}
Q_{\mathrm{DT}}(\xi) = \frac{1}{32\pi G l^3}\int d\phi \Bigg\{ & \left(1-\frac{1}{2m^2l^2}\right)(4l^2(T^+ f_{++}+T^- f_{--}) \\
& +\left(1+\frac{1}{2m^2l^2}\right)(T^+ + T^-)(f_{rr}-4l^2 f_{+-}) \\
& +\left(1+\frac{1}{2m^2l^2}\right)\, r\, \left[(T^+ + T^-)\left(h_{rr}-2l^2 h_{+-}\right)+2l^2\left(T^+ h_{++}+T^- h_{--}\right)\right]\Bigg\} \ .
\end{aligned} \tag{3.13}
$$

In the generic case $m^2 \neq -\frac{1}{2l^2}$, for the Brown-Henneaux boundary conditions, where the h-terms are absent, the result of Liu and Sun [47] is recovered.

Note that the linearly divergent pieces coming from the standard and the purely quadratic pieces, in eqs. (3.11) and (3.12), respectively, combine such that for the special case $m^2 = -\frac{1}{2l^2}$ they cancel out. It is worth pointing out that the second line of (3.13), which is a term of order one, also vanishes in the special case. This goes by hand with the fact that the combination $f_{rr} - 4l^2 f_{+-}$ is generically required to vanish by the field equations, except at the special case. This brings in the freedom to introduce the additional integration constant b in our black hole solution. Therefore, in the special case the charges are given by

$$
Q_{\mathrm{DT}}(\xi) = \frac{1}{4\pi G l}\int d\phi\left(T^+ f_{++}+T^- f_{--}\right) \ . \tag{3.14}
$$

Note that as this expression does not depend on the h-terms in the asymptotic conditions (3.7), which cannot be gauged away in general, they could be regarded as a kind of "gravitational hair".

The central charge can then be obtained from the variation of the charge (3.14) along an asymptotic symmetry, $\delta_{\xi_1} Q_{\mathrm{DT}}(\xi_2)$, evaluated on the AdS background, and it is found to be

$$
c_\pm = c = \frac{3l}{G} \ . \tag{3.15}
$$

For the special case, this result is in agreement with [47, 48] for BHT massive gravity with Brown-Henneaux boundary conditions. This is natural since according to the general theorems of ref. [49], as the central charge depends on the parameters of the theory and on the chosen background, its value is not expected to change for a relaxed set of asymptotic conditions that includes the standard asymptotic symmetries.

The mass of the black hole (3.1), measured with respect to an AdS background is given by

$$
M = Q_{\mathrm{DT}}(\partial_t) = \frac{1+\mu}{4G} \,, \tag{3.16}
$$

which does not depend on the integration constant b. Note that the mass of the BTZ black hole ($b=0$) for the BHT theory at the special case is twice the value obtained for GR.

3.4 Black hole thermodynamics

It is useful to express the metric of the Euclidean continuation of black hole (3.1) as

$$
ds^2 = l^2\left[\sinh^2\rho \, d\tau^2 + d\rho^2 + \frac{1}{4}\left((r_+ + r_-) + (r_+ - r_-)\cosh\rho\right)^2 d\phi^2\right] \,, \tag{3.17}
$$

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where

$$r = \frac{l}{2}\left[(r_+ - r_-)\cosh\rho + r_+ + r_-\right] \,, \tag{3.18}$$

excludes the region inside the event horizon, so that $0 \leq \rho < \infty$, and $0 \leq \tau < \beta$. The Hawking temperature is then given by the inverse of the Euclidean time period β, which is found requiring the Euclidean metric to be smooth at the origin

$$T = \frac{1}{\beta} = \frac{1}{4\pi l}\sqrt{b^2 l^2 + 4\mu} \,, \tag{3.19}$$

$$= \frac{r_+ - r_-}{4\pi l^2} \,. \tag{3.20}$$

The temperature vanishes for the extremal case ($b < 0$), for which the Euclidean metric (3.17) reduces to $H_2 \times S^1$, where H_2 is the two-dimensional hyperbolic space of radius l. Although the coordinate transformation (3.18) is ill-defined in this case, it is simple to show that $H_2 \times S^1$, as well as its Lorentzian continuation, $AdS_2 \times S^1$, solve the field equations (1.9) for the special case (1.12). This means that the near horizon geometry of the extremal black hole is given by $AdS_2 \times S^1$, and from eq. (3.17), it is amusing to verify that the parameter b, in eq. (3.3), can be identified with the period of the circle S^1.

As it is shown below, a spacetime of the form $R \times H_2$, which corresponds to a double Wick rotation of $AdS_2 \times S^1$ where the circle S^1 is unwrapped, yields an interesting solution.

3.4.1 Black hole entropy

The entropy of the black hole (3.1) can be obtained by Wald's formula [50]. Following, the conventions of [51], the entropy can be obtained from

$$S = -2\pi \int_{\Sigma_h} \frac{\delta L}{\delta R_{\mu\nu\alpha\beta}} \epsilon_{\mu\nu}\epsilon_{\alpha\beta}\bar{\epsilon} \,, \tag{3.21}$$

where L is the Lagrangian, and $\bar{\epsilon}$, $\epsilon_{\mu\nu}$, correspond to the volume form and the binormal vector to the space-like bifurcation surface Σ_h, respectively. Here $\epsilon_{\mu\nu}$ is normalized as $\epsilon^{\mu\nu}\epsilon_{\mu\nu} = -2$. For the BHT action one obtains

$$\frac{\delta L}{\delta R_{\mu\nu\alpha\beta}} = g^{\mu\alpha}\left[g^{\nu\beta} - \frac{2}{m^2}\left(R^{\nu\beta} - \frac{3}{8}g^{\nu\beta}R\right)\right] \,, \tag{3.22}$$

and for the black holes discussed here the binormal vector is given by

$$\epsilon_{\mu\nu} = -2\delta^t_{[\mu}\delta^r_{\nu]} \,. \tag{3.23}$$

Hence, the entropy of the black hole (3.1) is found to be

$$S = \frac{\pi l}{2G}\sqrt{b^2 l^2 + 4\mu} \,,$$

$$= \frac{1}{4G}\left(A_+ - A_-\right) \,, \tag{3.24}$$

where $A_\pm = 2\pi r_\pm$ corresponds, for $b < 0$, to the area of the event and the inner horizons.

Note that for the BHT theory at the special case $\lambda = m^2$, the entropy of the BTZ black hole ($b = 0$) is twice the one obtained from GR.

It is reassuring to verify that the mass computed form the Deser-Tekin approach and the entropy (3.24) fulfill the first law $dM = TdS$.

3.5 Gravitational solitons, kinks and wormholes

A different class of exact solutions of the field equations (1.9) for the special case $m^2 = \lambda$, can be obtained from a double Wick rotation of the black hole (3.1). The solution generically describes a gravitational soliton, and for the extremal case, corresponds to a kink that interpolates between different vacua. Performing a suitable identification, a wormhole solution in vacuum can also be obtained as a limiting case of the gravitational soliton.

This can be seen as follows: A smooth Lorentzian solution is obtained from (3.17), unwrapping the angular coordinate, making $\phi \to it$ and rescaling the Euclidean time as $\tau \to \frac{\beta}{2\pi}\phi$. The metric reads:

$$ds^2 = l^2 \left[-\frac{1}{4} \left((r_+ + r_-) + (r_+ - r_-) \cosh \rho \right)^2 dt^2 + d\rho^2 + \sinh^2 \rho d\phi^2 \right] , \tag{3.25}$$

where the range of the coordinates is given by $-\infty < t < +\infty$, $0 \leq \phi < 2\pi$, and $0 \leq \rho < +\infty$. In this case, the integration constants r_+ and r_- are no longer interpreted as horizons.

3.5.1 Wormhole

Note that for the case $r_+ = r_-$ the metric reduces to a static universe of negative spatial curvature, i.e., $R \times H_2$, where H_2 is of radius l. This spacetime can also be obtained from a double Wick rotation of $AdS_2 \times S^1$, but once the circle S^1 is unwrapped, the link with the gravitational hair given by the period of S^1 is lost.

A wormhole solution in vacuum can be constructed performing an identification of the hyperbolic space H_2 along a boost of its isometry group, parametrized by a constant. The metric then reads

$$ds^2 = -dt^2 + l^2 \left[dz^2 + \rho_0^2 \cosh^2 z d\phi^2 \right] , \tag{3.26}$$

where $-\infty < z < +\infty$. This spacetime then corresponds to the product of the real line with a quotient of the hyperbolic space of the form $R \times H_2/\Gamma$, (where Γ is a boost of $SO(2,1)$ parametrized by ρ_0 (see, e.g. [52])). The solution describes a wormhole in vacuum whose neck, of radius $\rho_0 l$, is located at $z = 0$ and connects two static universes of negative spatial curvature located at $z \to \pm\infty$.

Note that since the whole spacetime is devoid of matter, no energy conditions can be violated. The causal structure of the wormhole (3.26) coincides with the one of Minkowski spacetime in two dimensions, as it is depicted in figure 3.

3.5.2 Gravitational soliton

For the generic case, $r_+ \neq r_-$, after a suitable rescaling of time, the metric (3.25) depends on a single integration constant and can be written as

$$ds^2 = l^2 \left[-\left(a + \cosh \rho \right)^2 dt^2 + d\rho^2 + \sinh^2 \rho d\phi^2 \right] . \tag{3.27}$$

This spacetime is regular everywhere, whose Ricci scalar is given by

$$R = -\frac{2}{l^2} \frac{a + 3\cosh \rho}{a + \cosh \rho} , \tag{3.28}$$

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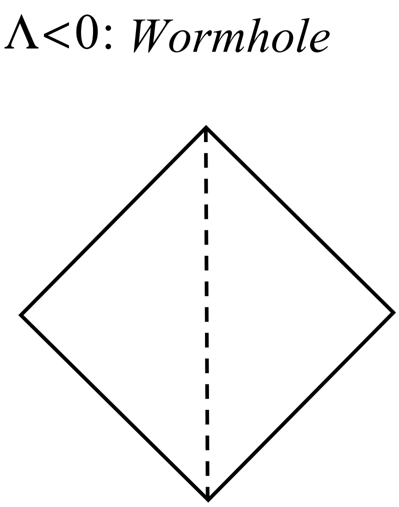

Figure 3. Causal structure of the wormhole. The dotted line corresponds to the location of the neck.

and describes a gravitational soliton provided $a > -1$. For $a = 0$, AdS in global coordinates is recovered. The soliton can then be regarded as a smooth deformation of AdS spacetime, sharing the same causal structure. This is an asymptotically AdS spacetime included within the set of asymptotic conditions given in eq. (3.7). This can be explicitly verified changing to Schwarzschild-like coordinates making

$$r \to l \sinh \rho \ ; \ t \to lt \,, \tag{3.29}$$

so that the metric reads

$$ds^2 = -\left(a + \sqrt{\frac{r^2}{l^2} + 1}\right)^2 dt^2 + \frac{dr^2}{\frac{r^2}{l^2} + 1} + r^2 d\phi^2 \ . \tag{3.30}$$

Thus, the mass of this solution measured with respect to an AdS background is easily obtained from the surface integral (3.14), and it is given by

$$M = -\frac{a^2}{4G} \ . \tag{3.31}$$

3.5.3 Gravitational kink

A gravitational kink can be obtained from the corresponding double Wick rotation of the black hole (3.1) for the extremal case. Making

$$r - r_+ = le^z \ ; \ t \to lt \,, \tag{3.32}$$

the metric reads

$$ds^2 = -(a + e^z)^2 \, dt^2 + l^2 \left[dz^2 + e^{2z} d\phi^2\right] \ , \tag{3.33}$$

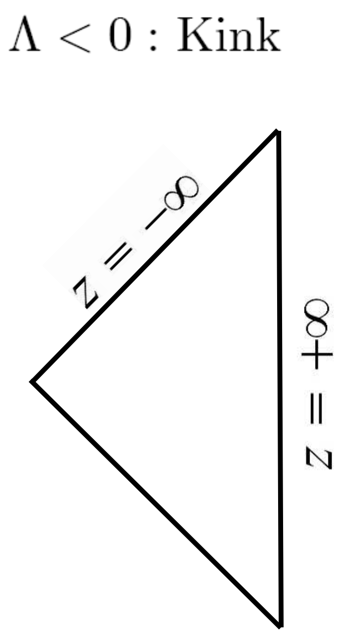

Figure 4. Causal structure of the kink, that interpolates between $R \times H_2$ ($z \to -\infty$) and AdS ($z \to +\infty$).

where $-\infty < z < +\infty$, and $a := \frac{r_+}{l} > 0$. For $a = 0$ one recovers the massless BTZ black hole. The kink interpolates AdS and a static Universe of negative spatial curvature ($R \times H_2$). The causal structure is shown in figure 4.

4 Positive cosmological constant

4.1 Black hole

For positive cosmological constant, $\Lambda := \frac{1}{l^2}$, a solution of Lorenzian signature of the field equations (1.9), at the special case $m^2 = \lambda$, is obtained from the Wick rotation of (2.1), through $\psi \to it$ and $\varphi = \phi$. The metric reads

$$ds^2 = -\left(-\frac{r^2}{l^2} + br - \mu\right) dt^2 + \frac{dr^2}{-\frac{r^2}{l^2} + br - \mu} + r^2 d\phi^2 \,, \tag{4.1}$$

where $-\infty < t < +\infty$, $0 \leq \phi < 2\pi$, and it describes asymptotically dS black holes provided the lapse function g_{tt} admits two positive real roots. In terms of the corresponding roots, $r_{++} > r_+$, the metric reads

$$ds^2 = -\frac{1}{l^2}(r - r_+)(r_{++} - r)dt^2 + \frac{l^2 dr^2}{(r - r_+)(r_{++} - r)} + r^2 d\phi^2 \,, \tag{4.2}$$

where the gravitational hair and mass parameters are given by

$$b = \frac{1}{l^2}(r_+ + r_{++}) > 0 \,, \tag{4.3}$$

$$\mu = \frac{r_+ r_{++}}{l^2} > 0 \,. \tag{4.4}$$

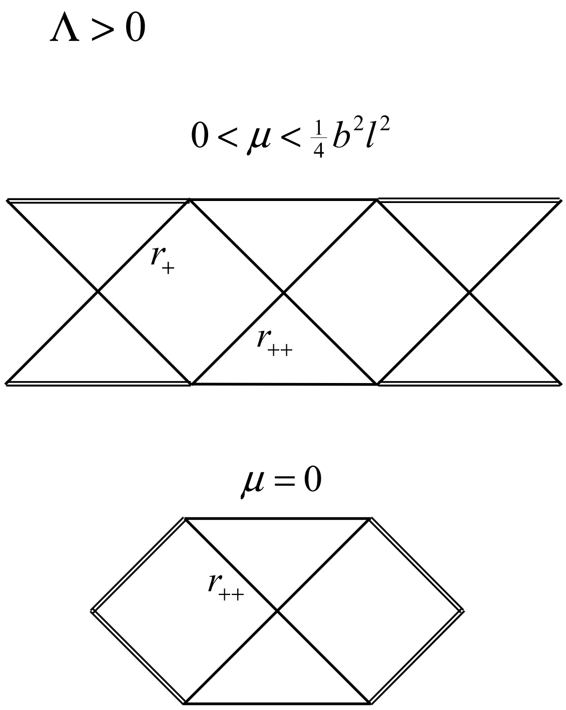

Figure 5. The causal structure of the black holes with positive cosmological constant.

Note that dS spacetime is recovered for $b = 0$, and $\mu = -1$.

The black hole (4.1) exists for the range

$$0 < \mu \leq \frac{1}{4} b^2 l^2 \,, \tag{4.5}$$

and possesses a spacelike singularity at the origin ($r = 0$) that is enclosed by the event horizon located at $r = r_+$, which is surrounded by the cosmological horizon at $r = r_{++}$. In the case of $\mu = 0$, there is a NUT at the origin, on top of the singularity which becomes null. The upper bound on the mass parameter is saturated at the extremal case, for which both horizons coincide ($r_+ = r_{++} = bl^2/2$). The causal structure is depicted in figure 5.

Note that the black hole exists due to the presence of the integration constant b. For a fixed mass parameter within the range (4.5), if the parameter b decreases, the event horizon radius increases, while the cosmological horizon shrinks.

Remarkably, since the Hawking temperatures of the event and of the cosmological horizon coincide, i.e.,

$$T_+ = T_{++} = \frac{r_{++} - r_+}{4\pi l^2} \,, \tag{4.6}$$

the solution can be regarded as a pair of black holes on dS. As a consequence, the Euclidean continuation of the black hole describes a regular instanton whose metric can be written as

$$ds^2 = l^2 \left[\sin^2\theta d\tau^2 + d\theta^2\right] + \frac{1}{4}\left((r_{++} + r_+) + (r_+ - r_{++})\cos\theta\right)^2 d\phi^2 \,, \tag{4.7}$$

where $0 \leq \tau < 2\pi$, and $0 \leq \theta < \pi$. The instanton (4.7) is homeomorphic to $S^2 \times S^1$ and it is obtained from (2.1) fixing the Euclidean time period according to $\beta = T_+^{-1} = T_{++}^{-1}$, and performing the following change of coordinates

$$r = \frac{1}{2}(r_{++} + r_+) + \frac{1}{2}(r_+ - r_{++}) \cos\theta \,, \tag{4.8}$$

which covers the region between the event horizon (located at the north pole, $\theta = 0$) and the cosmological horizon (located at $\theta = \pi$).

The temperature vanishes for the extremal case ($r_+ = r_{++}$), for which the Euclidean metric (4.7) reduces exactly to $S^2 \times S^1$, where the two-sphere is of radius l. Hence, this product space, as well as its Lorentzian continuation, $dS_2 \times S^1$, solve the field equations (1.9) for the special case (1.12). This means that the near horizon geometry of the extremal black hole is given by the three-dimensional analogue of the Nariai solution, where the parameter b, in eq. (4.3), can be identified with the period of the circle S^1.

Note that a three-dimensional Einstein Universe, $R \times S^2$, with a two-sphere of radius l is obtained from a double Wick rotation of $dS_2 \times S^1$ once the circle S^1 is unwrapped.

4.2 Gravitational soliton

A gravitational soliton can be obtained unwrapping and Wick-rotating the angle $\phi \to it$ and making $\tau \to \phi$ in the Euclidean black hole metric (4.7). The metric then reads

$$ds^2 = -\left((r_{++} + r_+) + (r_+ - r_{++})\cos\theta\right)^2 dt^2 + l^2\left[d\theta^2 + \sin^2\theta d\phi^2\right] \,, \tag{4.9}$$

so that the range of the coordinates is given by $-\infty < t < +\infty$, $0 \leq \phi < 2\pi$, and $0 \leq \theta < \pi$.

Note that for $r_+ = r_{++}$, this metric reduces to the static Einstein Universe $R \times S^2$; otherwise, after a suitable rescaling of time, the metric (4.9) depends on a single integration constant and reduces to

$$ds^2 = -\left(a + \cos\theta\right)^2 dt^2 + l^2\left[d\theta^2 + \sin^2\theta d\phi^2\right] \,, \tag{4.10}$$

describing a gravitational soliton provided $|a| > 1$.

4.3 Euclidean action

In the case of positive cosmological constant, the Euclidean black hole metric (4.7) coincides with the Euclidean continuation of the soliton in eq. (4.10). It is also worth pointing out that neither the Euclidean black hole nor $S^2 \times S^1$ have a boundary. Thus, it is simple to show that the Euclidean continuation of the action (1.7) evaluated on these solutions vanishes, i.e.,

$$I(bh) = I(S^2 \times S^1) = 0 \,. \tag{4.11}$$

This is to be compared with the value of the Euclidean action for the three-sphere of radius l (Euclidean dS space), given by

$$I(S^3) = \frac{l}{G} \,, \tag{4.12}$$

which allows to estimate the pair creation ratio [53].

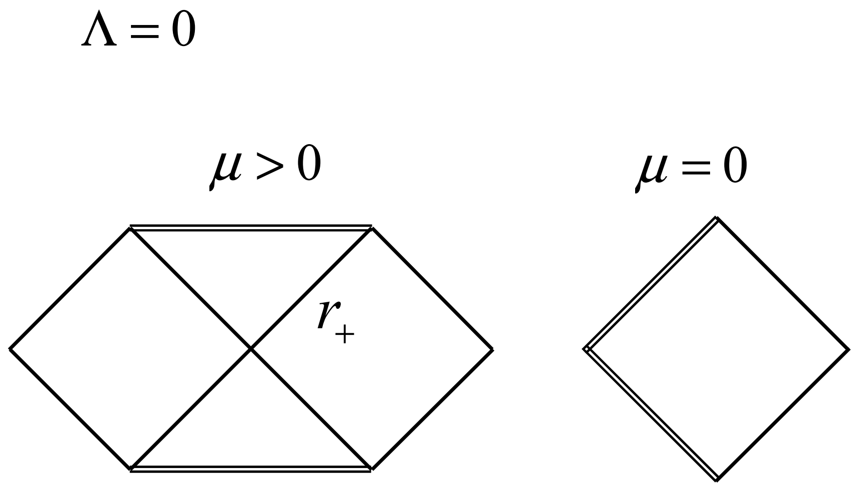

Figure 6. Penrose diagrams for the asymptotically locally flat black holes.

5 Vanishing cosmological constant

In the case of vanishing cosmological constant, the BHT action (1.7) at the special point $\lambda = m^2$ reduces to

$$I = \int d^3x\sqrt{-g}K\,, \tag{5.1}$$

which, as it has been recently shown in [30], enjoys remarkable properties. An asymptotically locally flat black hole solution of this theory can be obtained from the metric (2.1) making $\Lambda = 0$, and $\psi \to it$

$$ds^2 = -\left(br - \mu\right)dt^2 + \frac{dr^2}{br - \mu} + r^2 d\varphi^2\ . \tag{5.2}$$

This black hole possesses a spacelike singularity at the origin, which is surrounded by an event horizon located at $r_+ = \mu/b$, provided $b > 0$ and $\mu > 0$. In the case of $\mu = 0$, there is a NUT at the origin, on top of the null singularity. The causal structure is shown in figure 6.

A gravitational soliton can also be obtained from a double Wick rotation of (5.2), of the form $t \to i\phi$ and $\varphi \to 2it$. After a change of coordinates, given by $\rho = \sqrt{br - \mu}$, the spacetime possesses a conical singularity at the origin, which is removed for $b = 2$. The metric is then smooth everywhere and reads

$$ds^2 = -\left(\rho^2 + \mu\right)^2 dt^2 + d\rho^2 + \rho^2 d\phi^2\,, \tag{5.3}$$

with $0 \le \rho < \infty$.

As shown in [30] the degree of freedom associated with the massive graviton, at the linearized level, is captured by the h_{ti} component of the metric deviation. Note that for

GR, the graviton degrees of freedom cannot be excited in this way while keeping spherical symmetry. For our solutions (5.2) and (5.3) it is apparent that the massive graviton degree of freedom is switched off. It would be interesting to explore the existence of analytic solutions in the full nonlinear theory, for which the degree of freedom could be consistently switched on in the presence of the black hole (5.2) or the gravitational soliton (5.3).

One may also wonder about whether the asymptotically locally flat black hole (5.2), and the gravitational soliton (5.3) can be accommodated within a suitable set of asymptotic conditions at null infinity, along the lines of ref. [54].

6 Rotating solutions

The solutions discussed here can be generalized to the rotating case by means of an (improper) boost in the $t-\phi$ plane (For a discussion about this subject in three-dimensional General Relativity see, e.g. [55]). For instance, the rotating extension of the asymptotically AdS black hole (3.1) is given by

$$ds_a^2 = -NFdt^2 + \frac{dr^2}{F} + r^2\left(d\phi + N^\phi dt\right)^2 , \tag{6.1}$$

with

$$\begin{aligned}
N &= \left[1 - \frac{bl^2}{4\mathcal{G}}\left(1 - \Xi^{\frac{-1}{2}}\right)\right]^2 , \\
N^\phi &= -\frac{a}{2r^2}\left(4GM - b\Xi^{\frac{-1}{2}}\mathcal{G}\right) , \\
F &= \frac{\mathcal{G}^2}{r^2}\left[\frac{\mathcal{G}^2}{l^2} + \frac{b}{2}\left(1 + \Xi^{\frac{-1}{2}}\right)\mathcal{G} + \frac{b^2 l^2}{16}\left(1 - \Xi^{\frac{-1}{2}}\right)^2 - 4GM\ \Xi^{\frac{1}{2}}\right] ,
\end{aligned}$$

and

$$\mathcal{G} = \left[r^2 - 2GMl^2\left(1 - \Xi^{\frac{1}{2}}\right) - \frac{b^2 l^4}{16}\left(1 - \Xi^{\frac{-1}{2}}\right)^2\right]^{\frac{1}{2}} . \tag{6.2}$$

Here $\Xi := 1 - a^2/l^2$, and the angular momentum is given by $J = Ma$, where M is the mass (measured with respect to the zero mass black hole) and $-l < a < l$ is the rotation parameter. This spacetime is then asymptotically AdS and naturally fulfills the relaxed asymptotic conditions in eq. (3.7).

Depending on the range of the parameters M, a and b, the solution possesses an ergosphere and a singularity that can be surrounded by event and inner horizons. As for the nonrotating case, there are different branches of solutions according to the sign of the integration constant b. One can show that the temperature and the entropy are given by

$$T_a = \gamma^{-1} T , \tag{6.3}$$

$$S_a = \gamma S , \tag{6.4}$$

where $\gamma^2 = \frac{1}{2}\left(1 + \Xi^{-1/2}\right)$, and T, S correspond to the temperature and the entropy for the static case, respectively. For the rotating BTZ black hole ($b = 0$) the entropy becomes twice the one obtained in GR, i.e., $S = \frac{A_+}{2G}$. In the case of $b < 0$ the mass is bounded from below as

$$M \geq -\frac{b^2 l^2}{16G\Xi} \,.$$

This bound is saturated for the extremal case. Further details about the rotating case will be discussed elsewhere.

7 Discussion and final remarks

The black hole metrics of the form

$$ds^2 = -\left(-\Lambda r^2 + br - \mu\right) dt^2 + \frac{dr^2}{-\Lambda r^2 + br - \mu} + r^2 d\varphi^2 \,, \tag{7.1}$$

and the wormhole in eq. (3.26) were known to be solutions of conformal gravity in three dimensions [42]. Here it was shown that they are nice spacetimes, in the sense that they solve the field equations for theories beyond the one they were intended to. This is also the case for the rest of the solutions discussed here, including the rotating black holes (6.1), the gravitational solitons in eqs. (3.27), (4.10) and (5.3), the kink (3.33), the static universes of constant spatial curvature $R \times S^2$ and $R \times H_2$, as well as the product spaces $AdS_2 \times S^1$ and $dS_2 \times S^1$. Therefore, they are all solutions of the BHT field equations for the special case, $m^2 = \lambda$, even in presence of the topological mass term

$$G_{\mu\nu} + \lambda g_{\mu\nu} - \frac{1}{2m^2} K_{\mu\nu} + \frac{1}{\mu} C_{\mu\nu} = 0 \,, \tag{7.2}$$

where $C_{\mu\nu} := \epsilon^{\kappa\sigma\mu} \nabla_\kappa \left(R_{\sigma\nu} - \frac{1}{4} g_{\sigma\nu} R\right)$ is the Cotton tensor.

In the case of the black hole with positive cosmological constant (4.1), since the temperatures of the event and the cosmological horizons coincide, the solution can be regarded as a pair of black holes on dS, whose Euclidean continuation describes an instanton of vanishing Euclidean action. Thus, the pair creation ratio could be estimated from the value of the Euclidean action for S^3, given by l/G.

In the case of negative cosmological constant, the black hole and the gravitational soliton were shown to fit within the set of asymptotically AdS conditions given by (3.7), having a relaxed behavior as compared with the one of Brown and Henneaux. The asymptotic symmetries contain two copies of the Virasoro group, which is enlarged to a semi-direct product by the additional asymptotic symmetries associated to local shifts in the radial coordinate generated by (3.8). Note that this additional asymptotic symmetry can be used to eliminate the function h_{rr} appearing in the asymptotic conditions, so that for $h_{rr} = 0$ the asymptotic symmetry group reduces to the standard conformal group in two dimensions. Furthermore, this additional symmetry has vanishing charges, as it is apparent from eq. (3.14), which suggests that the gauge could always be fixed in this way. Nevertheless, since the remaining charges, as the mass, change nontrivially under shifts of the radial coordinate, this is not the case. This puzzling enhancement of the asymptotic symmetries could be related to the fact that the Deser-Tekin charges are constructed from the linearized theory. Indeed, there are known cases where nonlinear corrections are needed in order to obtain finite charges (as it occurs in the presence of scalar fields [6, 8]). Therefore, it would be desirable to explore this problem within a fully nonlinear approach. It is worth pointing out that this curious behavior may occur even for a nonlinear approach, as it has been observed for different classes of degenerate dynamical systems [58], even in classical mechanics [59], for which the rank of the symplectic form may decrease for certain regions within the space of configurations. Thus, around certain special classes of solutions, additional gauge symmetries arise, and hence the system losses some degrees of freedom.

It is also worth pointing out that further asymptotically AdS solutions with a different behavior at infinity exist for the BHT theory at the special point, as it is the case for the AdS waves recently found by Ayón-Beato, Giribet and Hassaïne [27]. A suitable set of asymptotically AdS conditions, containing the black hole (3.1) as well as the AdS waves, is such that the deviation with respect to the AdS metric (1.3) is supplemented by additional terms with logarithmic behavior. Fixing the asymptotic symmetry under local shifts in the radial coordinate, the asymptotic conditions that are invariant under the two copies of the Virasoro algebra, generated by (1.4), read

$$\begin{aligned} \Delta g_{rr} &= f_{rr}\, r^{-4} + \dots\,, \\ \Delta g_{rm} &= j_{rm}\, r^{-2}\log(r) + h_{rm}\, r^{-2} + f_{rm}\, r^{-3} + \dots\,, \\ \Delta g_{mn} &= j_{mn}\, r\log(r) + h_{mn}\, r + f_{mn} + \dots\,, \end{aligned} \tag{7.3}$$

where j_{+-} can be consistently switched off, and $j_{\mu\nu}$ depends only on the time and the angle, but not on r. One can also verify that the Deser-Tekin charges, once evaluated on the asymptotic conditions (7.3), reduce to the expression given in eq. (3.10), which depend only on f_{++}, and f_{--}.

For the BHT theory in the generic case $m^2 \neq \lambda$, black holes which are not of constant curvature, that depend on an additional integration constant that is not related to the mass exist, and they fulfill the Brown-Henneaux boundary conditions [56]. It is amusing to see that this behavior has a similar pattern as the one of the Boulware-Deser solution for the Einstein-Gauss-Bonnet theory. In the generic case, relaxed asymptotic conditions including logarithmic branches have also been studied in ref. [47].

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